**Perturbative Expansion of GF**

**GContour starting in |Ω0> at t = t0 with V­time-dependent(t) whenever**

Note since we’re starting in |Ω0> at time t = t0 (we’ll allow t0 to be non-zero for the sake of discussion), our time development operator will therefore be U(t,t0) = U0(t,t0)S(t,t0). And so the interaction picture operator will be given by A(t,t0) = U0†(t,t0)AU0(t,t0), and I’ll just call this A(t). Like with the distinct particles case, it doesn’t seem advantageous to work this out from an equation of motion approach. So we’ll just do the Wick thing. Like before, we need the four GF’s to make the expansion self-consistent:



I will call these four GContour collectively, for short. We will consider the operators to evolve according to the Hamiltonian,



where H0 is a time-independent bilinear Hamiltonian,



and V(t) is the interaction, possibly time-dependent. But in QFT, V(t) is typically not time-dependent, because the fundamental entities out there are the fields themselves, not some explicitly time-dependent potential field from nowhere. In order to use Wick’s theorem, we’ll presume |Ω0> to be an eigenstate of H0 or some linear combination thereof. Furthermore, that it is ‘homogeneous’ – i.e., not a macroscopic quantum state, like a Bose condensate. Furthermore that we’re in the thermodynamic limit, etc. As aforementioned, the excitation vacuum satisfies these constraints.

**Perturbative expansion for G>**

Let’s form G>.



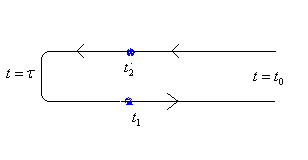
As covered in the single particle file, we can write this as:



where S would look like this:



and refers to the contour below:



We could make τ = ∞ if desired.

**Perturbative expansion for G<**

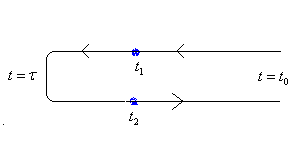
We start with,



And as covered in the single particle file, we can write this as:



Where the contour orders the times according to



Again we could make τ = ∞ if desired.

**Perturbative expansion for GC**

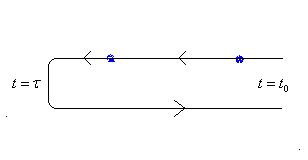
We start with



As in the single particle file, we can write:



where the contour orders the times according to:



and we can make τ = ∞.

**Perturbative expansion for GAC**

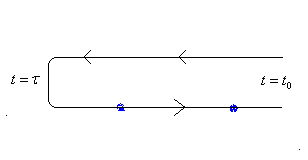
We start with



which we can write as:



with the contour…



and can make τ = ∞.

**General Diagrammatic Rules**

For example, let’s say we have:



for example, and we start off in |Ω0> at time t = t0. And we’re interested in any one of the GF functions in GContour (it’s more intuitive to state the rules in terms of the contractions, rather than the GF’s themselves). I’ll specialize to GF’s of the type where A and B are just (x) , though the procedure would work just as well for any bilinear in the creation / annihilation operators – for instance for too.



[my H is in terms of bosonic fields, but i’m allowing the fermionic possibility witht that ε in there, so we can conceptually apply it to fermion fields too] We can write all of these as:



where the times t and t´ are on the contours discussed above. Then we just expand,



(V is in interaction picture and implicit summation over indices) in a power series, where each term involves an integration that runs from t0 🡪 τ and back, and the two x’s in the GF are placed along the contour as shown above. Then we use Wick’s theorem to pick out contractions between the φα(xa,ta) and φα´(xb,tb) terms. In general we get:



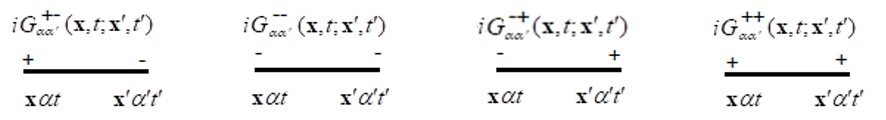
and we’ll recall the alternative notation G++ = GAC (anti-causal), G-- = GC (causal), G+- = G> (greater) and G-+ = G< (lesser). There is a shorthand notation that can be developed that’s very useful though. Can motivate it a little by considering the time-ordered correlation function.



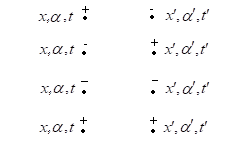
Might see that can write the expansion in terms of two vertices: one that time orders and another that anti-time orders. This leads to the Feynman rules below. I guess I’ll allow a time-dependent external field, even though that doesn’t typically happen. The interaction I’ll presume time-independent, though that case could also be handled here if of interest.

**Real Space Rules**

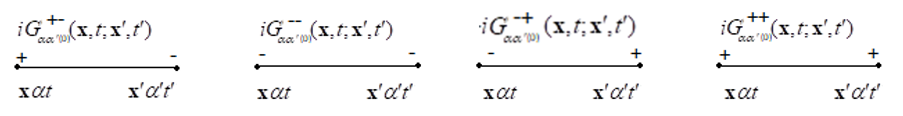
So we have our four GF’s. It’s easiest to write the expansion for the contour ordered contraction rather than the GF itself. In terms of the GF’s the contractions are (bold lines)



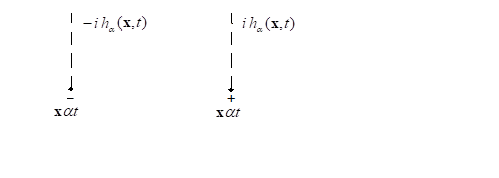
And the perturbative series works as follows. So we start with the external points. We represent the two arguments/operators in the GF we’re trying to get with external points. We use the first if we want G+-, the next for G-+, the next for G—and the last for G++.



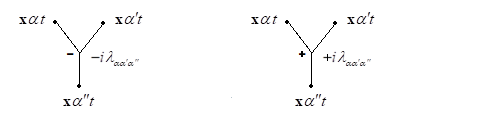
Then at each order of PT, we bring down the same number of vertices (discussed in a second) and connect with bare iGF’s. The bare (unperturbed) iGF’s are represented with non-bold lines:



The two S’s give rise to two vertices. The (-) vertex comes from the time ordered S, and the (+) vertex comes from the anti-time ordered vertex. The single particle potential would be:



and the interaction vertex, cubic in this example, would be:



The vertices are basically never position/time dependent themselves. Maybe there could be some index dependence. And we could move the x,t label on the vertex ends inward to the vertex itself, since they’re all the same. This is how it’s typically written.

**Topology**

Connect all topologically distinct, connected (meaning no bubbles, but end points can be disconnected from each other), diagrams together, associating each element in diagram with the indicated term.

**Equal time issues**

There are no equal time issues to be concerned about to my knowledge.

**Signs/Numerical Factors**

The topologically distinct diagrams show up in many different equivalent ways in the direct S-matrix expansion. And so we have to multiply each by the number of times it would appear in the expansion, divided by the 1/n! coming from the S-matrix expansion. The rules for calculating this number are, I think as follows. And it is probably necessary to stipulate the whatever λmm´m´´ is, it is symmetric w/r to interachange of any two index pairs, say for instance m with m´´, etc.



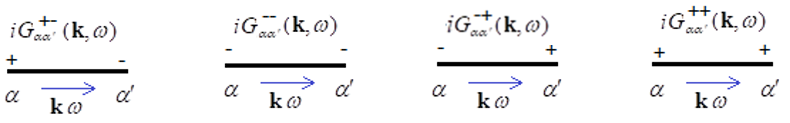
Can look to next file for more on this. But might have to pay particular attention to the last point here, as we wouldn’t expect (-) (+) vertices to be distinguishable, though this distinction wouldn’t apply in the next file.

**Sum**

Then sum/integrate over all internal positions/times/indices. There is a simplification to be had if we have a homogeneous isotropic medium. Then GF0 and GF are diagonal in their indices αα´. In that case, if we were to be interested in calculating G11, say, then the δαα´ attached to the GF would, along with the requisite index sum, end up fixing all GF’s in the diagram to that index G(0)11. So then we wouldn’t have to worry about any index sum per se’. Except in the case that we have closed loops. Then it seems to me to be similar to the fermion loop discussed in the identical particles file, and we’d have to multiply such loops by a factor of 3 (presuming GF11 = GF22 = GF33 which seems to be necessary in such case).

**Fourier Space Rules**

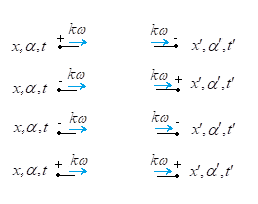
We can express the diagram in terms of Fourier components instead. Though, for finite time integrations, we’d have to put step functions on V(t) to allow integration over the entire real line. There is basically no point in doing this unless λ is position-independent. So for the sake of discussion I’ll presume so. Most of the following is justified in the Fourier transforms file. So are the Green’s functions, as usual:



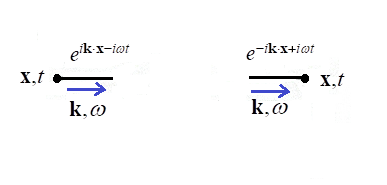
where these G’s are the Fourier transforms of the real space guys,



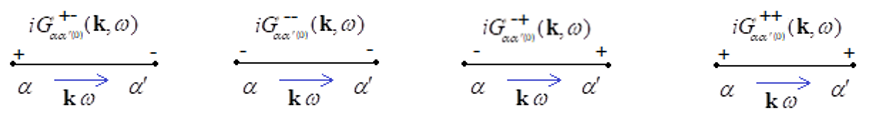
and then the external points are:



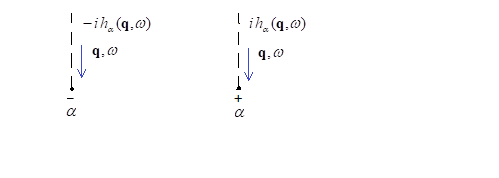
These carry the Fourier transform variable. And if we’re ultimately interested in the real space, time GF, then we must add the following factors to each:



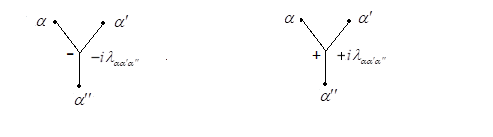
We connect these with the unperturbed GF’s:



Then the single particle potential is:



and the interaction vertex,



**Topology**

Connect all topologically distinct, connected (meaning no bubbles, but end points can be disconnected from each other), diagrams together, associating each element in diagram with the indicated term. And then we conserve energy/momentum at each vertex.

**Equal time issues**

None, again.

**Signs/Numerical Factors**

The same as above.

**Sum**

Then sum/integrate over all independent momenta (wavenumbers)/energies (frequencies), indices, including the external one if you want the real space GF.



and include the external momentum/frequency if want the real space GF.